

# Answer Sheet to the Written Exam

## Financial Markets

March 2012

In order to achieve the maximal grade 12 for the course, the student must excel in all three problems.

### Problem 1:

This problem focuses on testing part 1 of the course's learning objectives, that the students show "The ability to readily explain and discuss key theoretical concepts and results from academic articles, as well as their interpretation." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

(a) The textbook suggests on the top of page 166 that market quality arises when there is low variance of temporary price deviations around an efficient level — this is consistent with the volatility notion of the proposal. In the textbook's main theoretical models, limit orders are used by market makers who provide liquidity and quality — those market makers revise limit orders between incoming market orders, which might entail frequent limit order cancellations. If the proposal makes market making more expensive through this channel, market quality is likely to fall. Why, then, might excessive limit orders erode market quality? Perhaps some traders are trying to discover the fundamental asset value through the order book, and excess limit orders might obscure this process. Or some limit orders might be intended to manipulate the trading of order book observers.

(b) Draw on Chapter 3 in the textbook. Intuitively, each informed trader has some market power, but the greater the number of traders, the closer the market to a competitive situation (Cournot style). The total market order from the informed traders is then greater in size for given asset value. This implies that market makers need to adjust the price by a smaller amount for given order size — hence the market is more liquid.

(c) The difference between the two types of information seller is that the exchange can generate an income from granting access to trade. In the model of Cespa and Foucault (2011), the liquidity traders have inelastic demand. The group of speculators has more to gain when information is delayed, potentially allowing the exchange to profit more from granting access to speculators.

### Problem 2:

This problem focuses on testing part 2 of the course’s learning objectives, that the students show “The ability to carefully derive and analyze results within an advanced, mathematically specified theoretical model.” The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

(a) From the point of view of a market maker, the arrival of an informed trader implies that there is information in the market, as happens with chance  $\gamma$ . The conditional probability that the incoming trader is actually informed is  $\alpha$ . The total chance of an informed trader is thus  $\delta = \gamma\alpha$ . An uninformed trader of type  $t \in \{B, NT, S\}$  can arrive with conditional chance  $\beta_t$  in an uninformed market, and conditional chance  $(1 - \alpha)\beta_t$  in an informed market. The total chance of an uninformed trader of this type is thus  $(1 - \gamma)\beta_t + \gamma(1 - \alpha)\beta_t = (1 - \gamma\alpha)\beta_t$ .

(b) Market makers quote prices based on a belief about the distribution of the incoming traders. This belief is the same as in the original model, after the parameter change, so their strategy will be the same.

(c) I apologize for a mistake in this question. There is an  $\alpha$  missing from equation (1). Thus, the equation should be

$$\Pr(\text{Buyer}|\text{Info}) = (1 - \alpha)\beta_B + \alpha \Pr(V \geq a). \quad (1)$$

The mistake was soon reported by several students, and I announced the correction on the course homepage and via a mail to all students just over 8 hours into the exam. Grading the answers, it will be taken into account that attempts to answer this question may have taken extra time.

Again, the answer is based on the given information about the arrival of trader types. When there is information, a fraction  $\alpha$  of those will be informed, and those traders will buy when  $V > a$ , as happens with conditional probability  $\Pr(V \geq a)$ . Also, a fraction  $1 - \alpha$  will be uninformed, and buy with conditional probability  $\beta_B$ . This combines to expression (1). When there is no information, all traders are uninformed and buy with conditional probability  $\beta_B$ . Expression (2) follows.

(d) From Bayes’ rule it follows that  $\Pr(\text{Info}|\text{Buyer}) \geq \gamma$  if and only if  $\Pr(\text{Buyer}|\text{Info}) \geq \Pr(\text{Buyer}|\text{Noinfo})$ . Using (1) and (2), the latter condition is equivalent to  $(1 - \alpha)\beta_B + \alpha \Pr(V \geq a) \geq \beta_B$ , or,  $\alpha \Pr(V \geq a) \geq \alpha\beta_B$ , equivalent to the claimed  $\Pr(V \geq a) \geq \beta_B$  for  $\alpha > 0$ . For the case of replacing a buyer with a seller, equations (1)–(3) are only changed by replacing  $\beta_B$  with  $\beta_S$  and  $\Pr(V \geq a)$  with  $\Pr(V \leq b)$ . Likewise for an abstention, replace  $\beta_B$  with  $1 - \beta_B - \beta_S$  and  $\Pr(V \geq a)$  with  $\Pr(b < V < a)$ .

(e) For the uniform distribution, the mean and median are equal at  $\bar{V} = 1/2$ . Since  $a > \bar{V}$  (which is the median),  $\Pr(V \geq a) < 1/2$ . By the symmetric assumptions about the

buy and the sell side,  $\Pr(V \geq a) = \Pr(V \leq b)$ . Also,  $\Pr(b < V < a) > 0$ . Since  $1/2 = \beta_B = \beta_S$ , it follows from (d) that  $\Pr(\text{Info}|\text{Buyer}) < \gamma$ , and  $\Pr(\text{Info}|\text{Seller}) < \gamma$ . Also, when  $1 - \beta_B - \beta_S = 0$ , abstention happens with chance zero in an uninformed population, but with positive chance  $\alpha \Pr(b < V < a) > 0$  in an informed population — abstention then results in posterior  $\Pr(\text{Info}|\text{Abstention}) = 1$ , according to (3).

To discuss the result, note first that some informed traders rationally abstain when their signal indicates an asset value inside the bid-ask spread. If we assume that uninformed traders never abstain, it may follow that a lower trading volume should be interpreted as a sign of information in the trader population. On the other hand, some uninformed traders could abstain, and in fact informed traders might be expected to be more eager traders on average. Related to this, we have implicitly supposed that traders cannot arrive more frequently to the market makers, or place larger market orders, at times when the market is informed. In particular,  $1 - \beta_B - \beta_S = 0$  means that traders arrive at every opportunity in an uninformed market — perhaps not so realistic.

### Problem 3:

This problem focuses on testing part 3 of the course’s learning objectives, that the students show “The ability to apply the most relevant theoretical apparatus to analyze a given, new case-based problem.” The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

Below are some suggested applications of the course literature to this case. It is important to note that these applications have shortcomings which should be discussed.

- The text discusses a purpose of market regulation that is not directly mentioned in the textbook: protection from terrorists. As described, the goal of terrorism would be a form of lower price discovery, with a downward pricing bias, seeking to force stock prices to deviate further below some efficient level. This goal is easier to attain in a less resilient market, or a market of lower quality, notions well covered in the book.
- Somewhat akin to the transparency issues analyzed in Section 10.1 of the textbook, as well as by Cespa and Foucault (2011), it is alleged that terrorists may try to take advantage of a lack of transparency concerning their trading strategy. For instance, the small and mid-sized brokers might be able to profit from dual trading with access to this order flow.
- Connecting the two points, Section 10.1.3 suggests that liquidity and quality are improved when transparency is greater.